Smith Math 290 Fall 2010

Proof M-2

Accepted **Not Accepted**

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

- First due date **Thursday, November 4**
- Turn in your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- There is to be **no collaboration** on any aspect of developing and presenting your proof. Your only resources are: you, the course textbook, me, and pertinent discussions that occur **during class.**
- Follow the Writing Guidelines of the Grading Rubric in the course information sheet.
- Retry: Only use material from the relevant section of the text or earlier.
- Retry: Start over using a new sheet of paper.
- Retry: Restaple with new attempts first and this page on top.

"True eloquence consists in saying all that is necessary, and nothing but what is necessary." – La Rochefoucauld

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M-2 (Section FS)
Definition: If A is a square matrix of size m then we define $A^0 = I_m$, $A^1 = A$, and $A^{n+1} = A^n A$ for each $n \geq 1$. Further, if A is invertible, we define $A^{-n} = \left(A^{-1}\right)^n$

1. Suppose A and B are square matrices of size m and that A is non-singular. Use the principle of mathematical induction to prove that $\left(A^{-1}BA\right)^n = A^{-1}B^nA$ for every positive integer n.
2. Now suppose that B is also nonsingular and extend the previous result by proving the formula $\left(A^{-1}BA\right)^n = A^{-1}B^nA$ holds for every integer (positive, negative and zero).
3. Use your formula and the matrices $B = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$ and $A = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$ and the vector $\vec{x}_0 = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$ to compute $B^n\vec{x}_0$. What is the component by component limit of $B^n\vec{x}_0$ as $n \to \infty$?

Notes:

- In part 3, $A^{-1}BA$ should simplify to be a diagonal matrix.
- Recall the formula for powers of diagonal matrices (proven in class) and use it to compute B^n.
